Modeling Distributions

1. Lamar is shopping for a used car, and he's interested in determining the typical mileage on cars that are 3 or 4 years old. He looks at an online car buying site and compares the number of miles, in thousands, on 30 cars that are 3 years old and 30 cars that are 4 years old. His results are summarized below.

<table>
<thead>
<tr>
<th>Variable</th>
<th>N</th>
<th>Mean</th>
<th>StDev</th>
<th>Minimum</th>
<th>Q1</th>
<th>Median</th>
<th>Q3</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Four year old cars</td>
<td>30</td>
<td>56.68</td>
<td>17.82</td>
<td>23.60</td>
<td>47.80</td>
<td>54.70</td>
<td>64.50</td>
<td>100.30</td>
</tr>
<tr>
<td>Three year old cars</td>
<td>30</td>
<td>33.33</td>
<td>12.70</td>
<td>14.10</td>
<td>22.33</td>
<td>32.10</td>
<td>39.23</td>
<td>66.40</td>
</tr>
</tbody>
</table>

(a) The distribution of mileage on 4 year old cars follows a normal distribution. Estimate the number of four year old cars Lamar looked at that had been driven more than 42 thousand miles.

\[ Z = \frac{42 - 56.68}{17.82} = -0.82 \]

From Table A this is about the 20th percentile.

So about \(.8(30) = 24\) cars have been driven more than 42,000 miles

(b) Estimate the 60th percentile for mileage on the cars Lamar found that were four years old.

From Table A for the 60th percentile \( Z = 0.25 \)

So, \( Z(56) + \bar{x} = 0.25(17.82) + 56.68 = 61.1 \) thousand miles

(c) For 3 year-old car.

\[ Z = \frac{40 - 33.33}{12.7} \]

\[ Z = -0.26 \]

\( Z \) is 39.74 percentile

(c) One car that Lamar is interested in is four years old and has been driven 40 thousand miles. Another one is three years old and has 30 thousand miles on it. How does the number of miles on these cars compare, relative to other cars of the same age? Provide appropriate statistical calculations to support your answer.

Since \(-0.26 > -0.94\) this means that the 3 year old car has been driven more miles relative to other cars its age. The 3 year old car has been driven more than 39.74\% of cars its age while the four year old car only about 17.36\%. 

2. "Normal" body temperature varies by time of day. A series of readings was taken of the body temperature of a subject. The mean reading was found to be 36.5°C with a standard deviation of 0.3°C. If you wanted to convert the temperatures to the Fahrenheit scale, what would the new mean and standard deviation be? (Note: °F = °C(1.8) + 32).

\[
\text{Mean} = 36.5 (1.8) + 32 = 97.7°F
\]

\[
\text{Std Dev} = 0.3 (1.8) = 0.54°F
\]

(Note: Remember Std Dev, only Affects spread.)

3. A local post office weighs outgoing mail and finds that the weights of first-class letters is approximately Normally distributed with a mean of 0.69 ounces and a standard deviation of 0.16 ounces.

(a) What is the 60th percentile of first-class letter weights?

\[
Z = 0.25 \quad \mu + Z \sigma = 0.69 + 0.25(0.16) = 0.73 \text{ oz}
\]

(b) First-class letters weighing more than 1 ounce require additional postage. What proportion of first-class letters at this post office require additional postage?

\[
Z = \frac{x - \mu}{\sigma} = \frac{1 - 0.69}{0.16} = 1.94 \quad P(Z \leq 1.94) = 0.9738
\]

\[
1 - 0.9738 = 0.0262 \quad \text{About } 2.62\% \text{ of first-class letters require additional postage.}
\]

4. Old-fashioned mechanical alarm clocks were not very accurate about when the alarm went off. The density curve below describes the distribution of times a certain alarm clock went off. The scale on the x-axis represents when the alarm went off, in seconds, before (negative) or after (positive) the alarm was set to go off. What proportion of the time did the alarm go off within 10 seconds of the time it was set for? Shade the appropriate area on the graph to show how you found the answer.

\[
\frac{20}{60} = \frac{1}{3}
\]